AN IN-SERVICE TEACHERS' WORKSHOP ON MATHEMATICAL PROBLEM SOLVING THROUGH ACTIVITY-BASED LEARNING

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This paper describes an in-service professional development course in introducing creative approaches to teach mathematics for teachers from the private-funded schools in Java, Indonesia. The course focused on introducing teachers to problem-solving processes and skills, and samples of activity-based worksheet on problem solving through secondary school mathematics topics. The teachers' assignment presented as an end-of-course assessment and their feedback on this workshop are also presented in this paper.

Introduction

The current trend of mathematics education has been the increasing desire of educators and policy makers for a greater emphasis on thinking and problem-solving skills in the curriculum. The centrality of problem solving in the mathematics curriculum can be seen as a response to this increasing demand.

The mathematics curriculum throughout the world has been geared towards mathematical problem solving. The usefulness of mathematics lies in problem solving. As pointed out in the Cockcroft Report (1982), problem solving should be "at the heart of mathematics"(p.73). According to the report,

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> Mathematics is only 'useful' to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name 'problem solving'.
>
> (Cockcroft Report, 1982, p.73)

This was clearly echoed by the National Council of Teachers in Mathematics (NCTM, 1989) in the United States. According to the NCTM, mathematical problem solving should be integrated into the entire mathematics programme:

> Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but a process that should permeate the entire programme and provide the context in which concepts and skills can be learned.
>
> (NCTM, 1989, p.23)

A mathematics teacher must first be familiar with the problem solving heuristics and thinking skills required to solve mathematics problems. The teaching of mathematical problem solving to students naturally entails the teaching of the problem solving heuristics and thinking skills, which were systematically presented in Polya's (1971) classic book: *How to solve it*.

However, it is generally construed that problem-solving heuristics are useful for the higher achieving mathematics students. Research has not shown consistent results on the success of teaching problem solving heuristics to students and their resulting ability to solve non-routine problems without many constraints, see for example, Schoenfeld (1985). This brings us to one of the main concerns of mathematics teachers: the teaching of mathematical problem solving to the weaker students. The teaching of mathematics to weaker pupils has always been a challenging task faced by teachers, more so the teaching of mathematical problem solving to the weaker pupils. Anecdotal evidence from schools show that most current teaching practices used in schools do not address the problems in developing conceptual understanding faced by this group of pupils. The pupils in this group are mainly visual and kinaesthetic learners (Amir & Subramanian, 2007; Rayneri & Gerber, 2003). This group of pupils generally have low self-esteem and do not perform well in the school tests and examinations. As such, they are not usually selected for mathematics enrichment activities which are mainly sponsored by schools for the highly motivated students. Thus, their opportunities to wider exposure to "interesting" mathematics are further reduced. Due to these factors, these pupils appeared to show little interest in the subject; in classrooms, they tend not to be focused and invariably appear restless.

It is therefore necessary for a mathematics teacher to be familiar with mathematical problem solving processes (including the problem solving heuristics and thinking skills), the skill to design classroom activities which might excite kinaesthetic students in the classroom, and simultaneously introduce them to the mathematical problem solving processes of the mathematics curriculum. Thus, it becomes essential to equip teachers with skills to conduct activity-based problem solving lessons in mathematics classrooms.

The purpose of this paper is to describe an in-service mathematics professional development course for Indonesian teachers. The aim of this course was to familiarise the teacher participants with Polya's (1971) mathematical problem solving processes, including thinking skills and heuristics, and to design classroom activities for the students to acquire these processes. This paper also reports the teachers' participation during the course, the ideas of some sample activities used for the course participants, together with sample work and feedback provided by the course participants.

Description of the Course

The course was a three full-day workshop for teachers teaching in privately funded secondary schools from the entire Java during the summer vacation in 2006. Being an in-service workshop for teachers' professional development, the main objective of the course was to upgrade the teachers' mathematical content and pedagogical knowledge, update them on the problem-solving strategies in the teaching and learning mathematics, and also their skills in designing suitable activities to engage their pupils in acquiring mathematical problem solving heuristics.

The teachers attended the course either on a voluntary basis or were nominated by their schools. There were altogether twelve participants for the course, all of whom were teaching in different privately-funded schools in different parts of Java, Indonesia.

Out of all the participants, eleven of them were secondary teachers, while one of the participants was teaching in a primary school. All the secondary school participants signed up for the course on a voluntary basis, while the primary school teacher participant was nominated to attend the workshop by her school principal for her "professional enrichment". At the beginning of the course, she was already fully aware that this course was originally intended primarily for secondary school teachers, and hence might not have direct relevance to her immediate classroom teaching.

Principles in the Workshop Design

In designing this workshop, (i) students' self-esteem in mathematics achievement and (ii) social interaction in a typical classroom setting were taken into consideration besides being mindful of the need to make the participants aware of problem solving heuristics and skills and techniques in designing classroom activities.

Recent studies on mathematics self-concept show that the use of hands-on activities among students on mathematics tasks, rather than the traditional chalk-and-board teaching, could be instrumental in enhancing the students' self-esteem in the mathematics classroom (see for example, Lui, Liu, Lam & Toh, 2004). Through a series of small manageable steps geared towards achieving the lesson objectives, students could be led to build up their confidence in the subject. This principle was emphasised repeatedly during the workshop.

The focus on activity-based learning through student-student interaction was another emphasis of this workshop. Burns (1990) proposed that social interaction is one of the key points in the process of pupils learning mathematics. The more opportunities pupils have for social interaction with their peers, parents and teachers, affords the pupil's more viewpoints. Confronting the pupils with the thoughts of others provides the pupils with different perspectives on their own ideas, stimulating them to think through their own viewpoint (Burns, 1990).

In the workshop, some principles and ideas of incorporating cooperative learning in classrooms were also shown to the participants. For example, the participants were introduced to the four factors to consider in carrying out cooperative learning in classrooms as proposed by Leikin and Zaslavsky (1997): (1) small group size of about four to six members each; (2) learning tasks must ensure the pupils depend positively on each other; (3) learning environment must offer all members an equal opportunity to interact; and (4) each member is accountable for the group progress. All these were discussed in the context of the mathematics activities.

In this workshop, various heuristics were chosen as points for discussing the content of the workshop. Some of the following heuristics as proposed by Polya (1971) and illustrated by Foong (2006) were used:

1. Use models
2. Draw diagrams
3. Make a systematic list
4. Look for patterns
5. Guess and check
6. Simplify the problems
7. Work backwards

Rationale of the Course

This three-day workshop did not attempt to be exhaustive in the coverage of the entire secondary school mathematics curriculum; rather the workshop was selective in the coverage of the material. The workshop was not meant so much as to prepare an entire set of ready-made materials to be executed in local classroom context, but rather to engage the teacher participants in designing activities for the classroom, to introduce problem-solving heuristics and make learning of mathematics more meaningful, and at the same time to encourage the use of cooperative learning strategies. The two main topics selected for the workshop discussion were (1) counting and arithmetic; and (2) algebra.

Counting, also known as "arithmetic", is an area of mathematical problems that can be found in the most elementary mathematics curriculum to the rather advanced undergraduate course (Koh & Tay, 2007). Through counting and arithmetic, several problem-solving heuristics can be introduced, such as drawing diagrams, solving simpler problems, making systematic list and looking for patterns.

Algebra was identified as another main area where students generally have learning difficulties and misconceptions (Lee, 2006). In particular, studies have shown that students invariably find algebra more abstract in nature since algebra is mainly written in symbols (Lee, 2006; Usiskin, 1988).

In short, the rationale in designing the worksheet activities can be summarised as follows:

1. to make the learning of the abstract topics [algebra] more meaningful by using concrete objects to demonstrate and motivate whenever possible;
2. to introduce problem solving heuristics through small manageable steps for the average mathematics students to lead them to higher order thinking questions; and
3. to capitalise on the positive results of cooperative learning to further enhance students' learning.

Content of the Workshop

This section outlines some sample activities that were introduced in the three-day workshop.

Sample Activity One. To begin with an activity of applying the heuristics and thinking skills, the participants were introduced to the following problem:

> **Problem:** *How many squares are there in an 8 x 8 square?*
> (Lee, 2006, p.391, Q4).

The participants were guided to apply the various heuristics (solve simpler related problems, look for patterns, guess-and-check, make conjecture, verify answers) to solve non-routine mathematics problems. Through group activity, the participants were led to extend the problem to other problems with differing levels of difficulties:

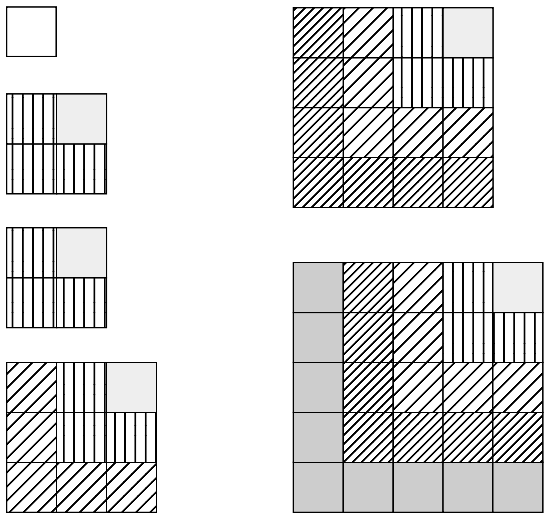
1. How many rectangles are there in an 8 x 8 square board?
   (Note to the teacher participants: A square is a special rectangle)
2. How many cubes are there in an 8 x 8 x 8 cube?
3. How many cuboids are there in an 8 x 8 x 8 cube?
   (Note to the teacher participants: A cube is a special cuboid)

*Sample Activity Two***.** In the next task on the Tower of Hanoi (with discs of unequal sizes), which is a well-known problem, cooperative learning strategies (Burns, 1990) were infused into solving this problem in an activity-based learning setting. Let T_n be the shortest number of steps in solving the problem of the Tower of Hanoi with n discs of unequal sizes. The participants were led to obtain $T_n = 2^n - 1$ by inductive reasoning and to obtain the recurrence relation $T_{n+1} = 2T_n + 1$ by deductive reasoning.

*Sample Activity Three***.** The participants were also introduced to the process of designing a problem gazing worksheet, and of simplifying a relatively challenging task, one that involves visualisation (see Figure 1), into one consisting of several manageable steps that can be used to guide the students to apply the problem solving heuristics.

Sums of Squares II

$3(1^2 + 2^2 + \cdots + n2) = (2n + 1)(1 + 2 + \cdots + n)$



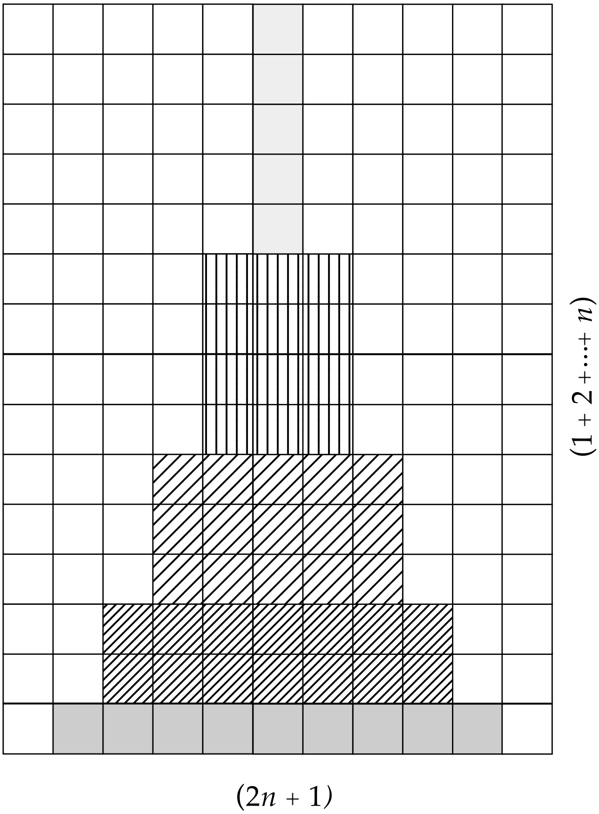


Figure 1. Sample of an activity for participants to design problem solving worksheet (Nelson, 1993, p. 78).

The instructor also used several other visualisation tasks with varying level of difficulties from Nelson (1993). These tasks were used for the purpose of designing tasks with several manageable steps.

*Sample Activity Four***.** The participants were introduced to different approaches that was aimed at making the processes of expansion and factorisation of algebraic expressions and the solving of algebraic equations more concrete through a series of hands-on activities to be used in class.

For example, factorisation of quadratic expressions can be transformed into activities involving the construction of rectangles. These ideas can be found in Toh (2006, pp. 57 - 60) and Lee (2006, pp. 49 - 53).

There are several ways of teaching the solving of algebraic equations: (1) as a way of interpreting what the solution to the equation means (see Lee, 2006; Toh, 2006); or (2) as the process of balancing two sides of a balance. The participants were shown these different ways of teaching the solution of algebraic equations, and how hands-on activities can be designed to make the process of solving algebraic equations more meaningful to their students.

Participants' Response During the Workshop

It could be seen that the participants were actively engaged in the activities throughout the course. Further, it was observed that several participants were actively trying to design and improve the activities so that they can be used in their own classrooms.

One of the participants told the instructor that he would like to try some of these activities in his own classroom. However, due to time constraints, he would like to try only some of the simpler activities.

Another participant explained that since algebra always posed difficulties for her students she would like to try some activities on algebra on her students. She also explained that to incorporate *all* such activities into her curriculum time would be unrealistic.

The participant from the primary school provided feedback that some of the activities, especially the pattern gazing type, were suitable for her students, after these activities were further simplified.

Products of the Participants

Towards the end of the third day of the workshop, the participants were asked to work in groups of four to either contribute useful ideas or design material that can be used as suitable activities on the various algebra topics for their teaching in the classrooms.

The participants were divided into three groups and were arbitrarily labeled as Groups A, B and C for the discussion here. The teacher participants presented their ideas in Bahasa Indonesia, which was translated by one of the teacher participant who was bilingual in both English and Bahasa Indonesia.

Group A expressed that it was difficult to teach binomial expansion to secondary school students. The students had difficulty in writing down the expansion of $(a+b)^3$. The participants from this group used the technique of physical interpretation of factorizing quadratic expressions and its relation to the different ways of counting rectangles (see Lee, 2006, p32) to include the third dimension algebraic expansion by designing a hands-on activity on the counting of cuboids.

Group B similarly agreed that it was difficult to teach students to remember the various formulae for algebraic expansion and factorisation. The members of the group then designed a hands-on activity for students which involved counting cuboids to derive the factorization formula $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$. The idea of the counting is illustrated in Figure 2.

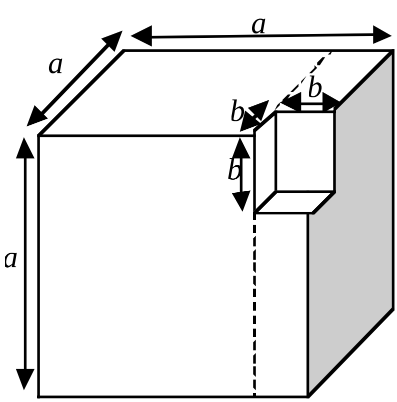


Figure 2. Concrete cuboids to illustrate the algebraic expansion of $a^3 - b^3$.

Group C deliberated on a pattern gazing worksheet involving the exploration of the Fibonacci sequence $\{u_n\}$, which satisfies the identity

$$u_{n+2} = u_{n+1} + u_n$$

where $u_1 = 1$ and $u_2 = 1$. They made many interesting observations about this sequence of numbers. For example, one part of the activity designed by this group was focused on guiding students to observe the parity of the last digits of this sequence. See Table 1.

Table 1

The Parity of the Terms in the Fibonacci Sequence

n	1	2	3	4	5	6	7	8	9
u_n	1	1	2	3	5	8	13	21	34
Parity	Odd	Odd	Even	Odd	Odd	Even	Odd	Odd	Even

Through discussion the group was able to provide a *deductive* argument on why the parity of the numbers occurred in this sequence. They observed that this was due to the fact that the sum of two odd numbers is even while the sum of one odd and one even number is odd.

Course Evaluation by the Participants

Towards the end of the third day of the course, a survey was conducted regarding the course. In this section the results of the survey is presented.

Eight out of the 11 participants indicated that they had improvement in their content knowledge after attending the course. Six of the participants graded their level of knowledge as Average – Good (Before and After Training) while two of the participants graded their level of knowledge as Good – Very Good (Before and After Training).

Table 2 below presents the participants' general perception of the three-day workshop.

Table 2
Participants' Perceptions of the Course

Question	Yes	No	Elaboration
The programme met the objective	11	-	
The topics were adequately covered	10	1	
The programme is relevant to developmental needs	10	1	Need for the primary school material
I will recommend this course to others	11	-	

Table 3 below presents the number of participants' perception of the feasibility of implementing the activities discussed during the three-day workshop. The topics of the workshop were divided according to the topics as shown in Table 3 below.

Table 3

Participants' Perceptions of the Feasibility of Implementation in Their Classrooms

Question	Number of Responses
The most significant subject learned	
a. Arithmetic	5
b. Algebra	5
c. Higher Algebra	6
d. Algebraic Factorisation	4
Subject easy to implement:	
a. Generalise Mathematical Formula	2
b. Algebraic Factorisation	2
c. Arithmetic	6
d. Algebra	3
e. Higher Algebra	1
Subject difficult to understand or implement	
a. Exponential Function	3
b. Polynomial Function	2

Generally, most of the participants found that the workshop met the instructional objectives, the topics were adequately covered and that the programme was relevant to their developmental needs, with the exception of one participant teaching in a local primary school. While the workshop attempted to discuss materials from a wide range of topics in the curriculum, most of the participants still indicated that they have learnt most from algebra and arithmetic topics. Most participants found the activities on arithmetic discussed in the workshop most easily implemented in actual classroom settings. There were also indications that the content that was most difficult came from the chapters on polynomial and exponential functions. The participants felt that it was difficult to design engaging activities that could capture the interest of their students on these more abstract topics in mathematics.

Discussion and Conclusion

As the current trend of mathematics education is towards mathematical problem solving and non-routine mathematical tasks, the use of traditional approach of drill-and-practice alone might not be sufficient. A mathematics teacher must possess both the pedagogical skills and sufficient mathematical content knowledge (Toh, Chua & Yap, 2007), which any teacher training institute needs to take note. While it is usually assumed that trainees who are enrolled into the teacher training institutes are likely to have sound content knowledge, with the changing curriculum emphasis and perhaps with wider intake of trainee teachers, this assumption needs to be reconsidered (Toh et al., 2007). Perhaps more effort should be taken by the teacher training institutes to allow opportunities for trainee teachers to improve their mathematical content knowledge.

Throughout the three-day workshop, it was observed that the level of enthusiasm of the participants was high and that the teachers were generally excited over the new approaches of teaching mathematics, which incorporated problem-solving heuristics and activity-based learning.

From the workshop it was also observed that there were different levels of ease in teaching a topic differently to fit the new paradigm of the mathematics curriculum. Generally, the teachers found it easier to adopt an activity-based problem solving approach in handling the more elementary topics (in this workshop, arithmetic) and more difficult to handle the more abstract topics. In this workshop, the participants indicated polynomial and exponential functions, or functions in general were more difficult. Based on this observation it can perhaps be suggested that in-service courses for teachers could be planned to focus on specific topics in the mathematics curriculum. These courses can be designed with the aim of allowing the teacher participants to acquire skills and knowledge in designing engaging activities based on mathematical problem solving for the specific topics. This would then allow the teachers to improve their pedagogical content knowledge in the areas which they are less competent.

Note: This paper only expresses the opinion of the author, who is also the instructor for the course. It does not express the views of the respective authorities.

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